[4] Brown, C. E. and Michael, W. H., Jr., "On slender delta wings with leading edge separation," NACA Langley Research Center, TN-3430 (April 1955).

[5] Jones, R. T., "Properties of low aspect ratio pointed wings at speeds below and above the speed of sound," NACA Langley Research Center, Rept. 835 (1946).

[6] Squire, L. C., "Pressure distributions and flow patterns on some conical shapes with sharp leading edges and symmetrical cross sections at $M = 4.0$," (Brit.) Aeronautical Research Council R & M 3340 (June 1962).

[7] Squire, L. C., "Pressure distributions at $M = 4.0$ on some 'flat' delta wings," (Brit.) Roy. Aircraft Estab., TN Aero 2865 (February 1963).

[8] Drougge, G. and Larson, P. O., "Pressure measurements and flow investigation on delta wings at supersonic speed," Aeronaut. Res. Inst. Swed. FFA Rept. 57 (November 1956).

Thrust Deflection for Cruise

W. H. Krase*
The Rand Corporation, Santa Monica, Calif.

Introduction

THE use of thrust deflection to improve cruise range is surely an old idea and one which has been investigated numerous times. But it may be particularly applicable to hypersonic airbreathing vehicles because of the combination of two technical factors: the moderate lift/drag ratios of hypersonic cruise vehicles and the low gross-thrust/ram-drag ratios of hypersonic airbreathing power plants. Low lift/drag ratios are known to make thrust deflection more effective, but the effects of gross-thrust/ram-drag ratio on thrust deflection have not, to the author's knowledge, been discussed. It is the primary purpose of this note, therefore, to indicate that hypersonic airbreathing vehicles may benefit substantially from thrust deflection.

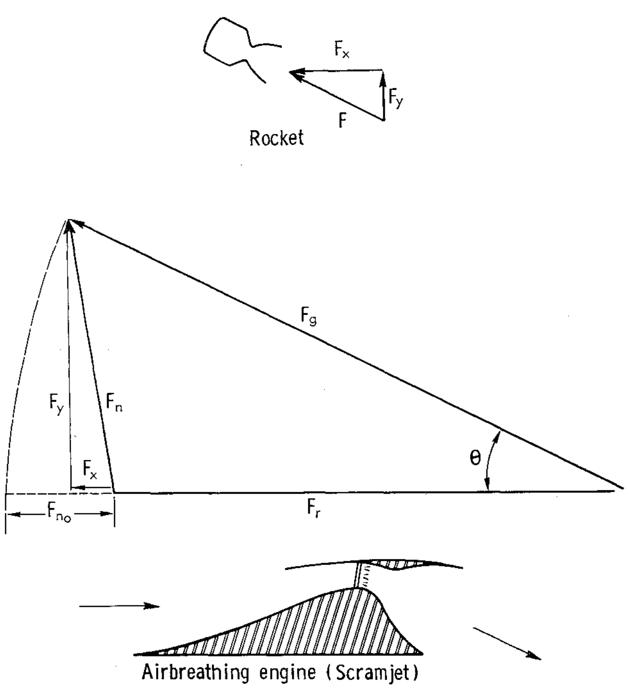

Fig. 1 Schematic of deflected rocket and airbreather.

Received October 31, 1966; revision received January 11, 1967. [3.02, 4.03]

* Engineer. Member AIAA.

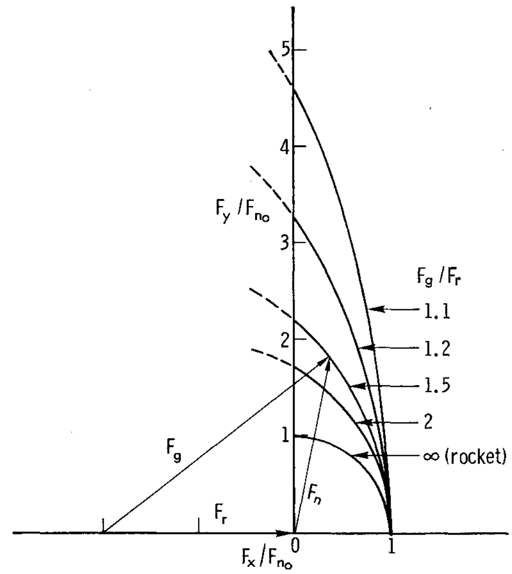

Fig. 2 Construction of thrust-component curves.

Discussion

The gross-thrust/ram-drag ratio (F_g/F_r) for a hypersonic airbreathing power plant may easily be as low as 1.1 at Mach 10 or 12, and, although estimates are quite uncertain, it tends to become still lower at higher speeds. Since this ratio is close to unity for ramjets or supersonic-combustion ramjets the vertical component of thrust, for a given deflection angle, is larger for these engines than for a rocket power plant. Figure 1 contrasts rocket and airbreathing power plants having the same undeflected thrust under the same nozzle deflection angle.† The essential point is that the gross thrust, which can be much larger than the net thrust in an airbreather, is the part that is deflected.

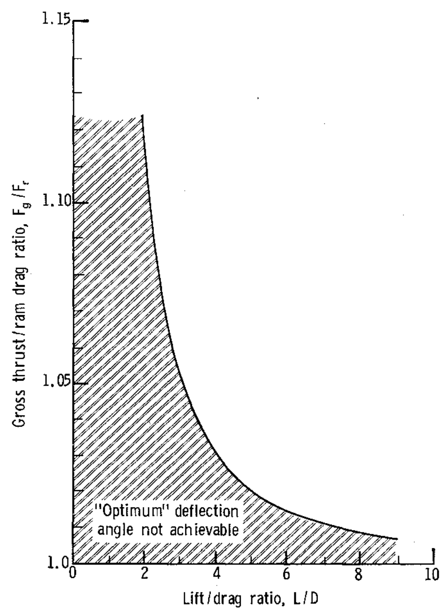

Fig. 3 Combinations of L/D and F_g/F_r for which the "optimum" deflection angle of Eq. (1) can be reached.

† A sketch like the vector diagram for the airbreather of Fig. 1 was given in a paper by G. S. Schairer, "Looking ahead in VSTOL," at the Eighth Anglo-American Aeronautical Conference, London, in 1961. Although his discussion made it clear that the features considered here were appreciated, they were not quantified and were not relevant to the subject under discussion there.

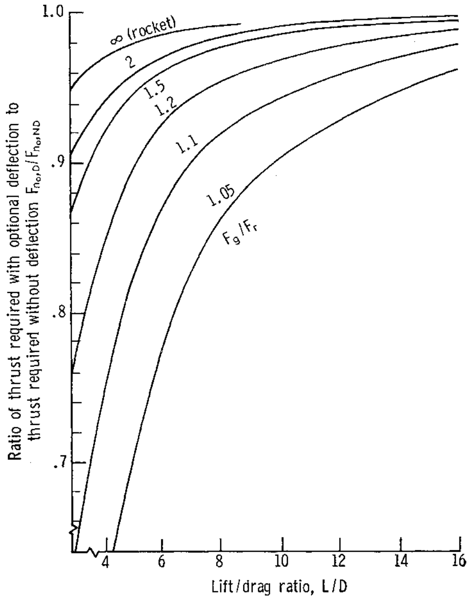

Fig. 4 Thrust required with deflection (at constant L/D).

Because power-plant performance is most conveniently and conventionally specified for conditions of no thrust deflection, the achievable thrust components will be normalized here by the undeflected net thrust (F_{no}) of a power plant having the same capture area and thermodynamic parameters. Figure 2 shows that the net-thrust vectors for any given gross-thrust/ram-drag ratio terminate on a circle arc whose radius, on a plot of (F_y/F_{no}) vs (F_x/F_{no}), is

$$\frac{F_g}{F_{no}} = \frac{F_g/F_r}{(F_g/F_r) - 1}$$

centered to the left of the origin.

Thrust deflection could be used in a number of ways, none of which will be analyzed completely here. It could be used to decrease wing size and weight at a constant altitude or to increase the altitude of a given vehicle. Here, we will assume that the vehicle weight, speed, shape, and L/D are fixed and then find the minimum thrust required when thrust deflection is used. Thrust deflection under these conditions results in operation at higher altitude and, for airbreathing power plants, requires a larger capture area (or larger corrected thrust), although the thrust and fuel flow are decreased. The use of thrust deflection tends to increase engine weight because of the higher capture area required, but the engine-weight increase would be offset at least in part by the reduced aerodynamic-heating load on the complete vehicle as well as the engine. However, assessment of these effects is beyond the scope of this paper. Here we will determine only the reduction in thrust and fuel flow, and by implication, the increase in range, achievable by thrust deflection, recognizing that engine weight or other design factors may modify the results. No attempt has been made to discuss stability or trim with thrust deflection.

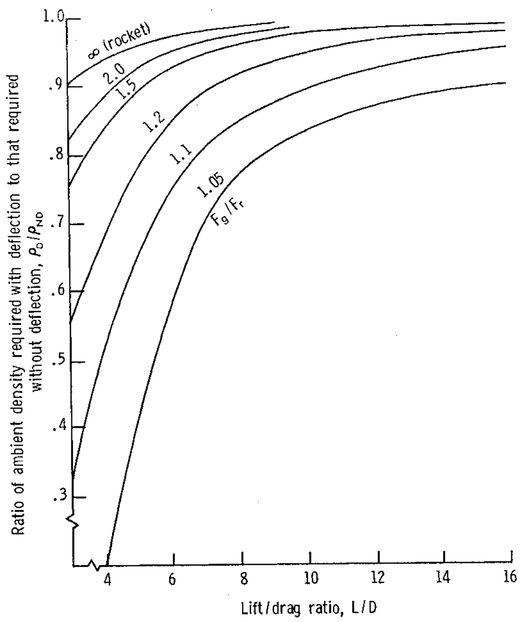

Fig. 5 Ambient density required with deflection (at constant L/D).

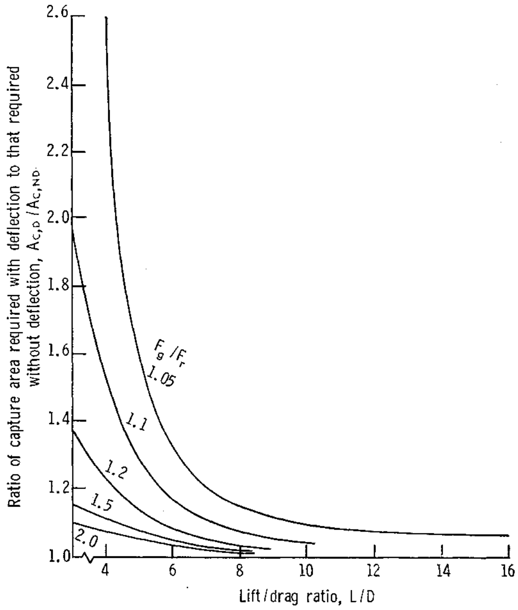

Fig. 6 Capture area required with deflection (at constant L/D (and same thermodynamic parameters.

The optimum nozzle deflection angle for a rocket under the present simplified rules is given by

$$\tan\theta_{\text{opt}} = \frac{1}{(L/D)} \tag{1}$$

The same rule can be shown to apply for an airbreathing engine: A force balance gives

$$D = F_g \cos\theta - F_r \tag{2}$$

$$W = L + F_g \sin\theta \tag{3}$$

Combining these equations, solving for W, and dividing by $F_g - F_r$ (which equals F_{no}) gives

$$\frac{W}{F_{no}} = \frac{L/D}{(F_g/F_r) - 1}\left[\frac{F_g}{F_r}\cos\theta - 1\right] + \frac{F_g/F_r}{(F_g/F_r) - 1}\sin\theta \tag{4}$$

Differentiating with respect to θ and setting the derivative equal to zero gives Eq. (1). But with airbreathing engines this optimum angle cannot be reached for certain combinations of L/D and F_g/F_r, because at low L/D the "optimum" angle may imply a negative drag (and lift). The maximum deflection angle (which gives zero drag), from Eq. (2), is given by

$$\cos\theta = \frac{1}{F_g/F_r}$$

Figure 3 shows the combinations of L/D and F_g/F_r for which the optimum angle of Eq. (1) is achievable. The line on Fig. 3 corresponds to zero drag and lift. As it is approached from the right, the required capture area increases very rapidly. Even if the optimum angle of Eq. (1) cannot be achieved, a substantial decrease in thrust is possible and can be found by applying Eq. (4).

Figure 4 shows the reduction in thrust achievable using the optimum deflection angle. Only cases for which an optimum thrust can be found are shown, and some of these are doubtless impractical also, as judged by the ratio of engine capture areas required. Moreover, it is not clear that required thrust should be minimized in a vehicle design; other considerations such as engine weight and aerodynamic heating also have an important bearing on vehicle design. But in any case, Fig. 4 indicates that rather substantial decreases in thrust can be realized by using thrust deflection. Figure 4 shows the minimum thrust/weight ratio achievable with thrust deflection, normalized by the thrust/weight ratio required for an undeflected installation as a function of L/D and F_g/F_r. Figure 5 shows the ambient-density ratio required for flight at the conditions of Fig. 4 (a measure of the altitude increase with deflection), and Fig. 6 shows the ratio of the engine capture area required with deflection to that required without deflection (assuming the same thermodynamic parameters).

The range increase that can be achieved with thrust deflection, under the present assumptions and neglecting possible engine-weight increases, would be given by the reciprocal of the thrust ratio in Fig. 4. Thus a vehicle with an L/D of 4, powered by an airbreathing engine with $F_g/F_r = 1.1$ and using thrust deflection of 14° would have about 34% greater range, would fly at an altitude about 15,000 ft higher, and would require a capture area 52% larger than a vehicle without thrust deflection.

Even at supersonic-transport cruise conditions, with $L/D \cong 8$ and $F_g/F_r \cong 1.2$, a thrust decrease and range increase of about 4% is possible with a deflection angle of 7.1° The increased altitude would also tend to decrease the sonic boom somewhat.